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Long time dynamics of von Karman evolutions with thermal effects *

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ABSTRACT: This paper presents a short survey of recent results pertaining to stability and long time behavior of von Karman thermoelastic plates. Questions such as uniform stability - and associated exponential decay rates for the energy function, existence of attractors in the case of internally/externally forced plates along with properties of attractors such as smoothness and dimensionality will be presented. The model considered consists of **undamped** oscillatory plate equation strongly coupled with heat equation. There are no other sources of dissipation. Nevertheless it will be shown that that the long-time behavior of the nonlinear evolution is ultimately **finite dimensional** and "smooth". In addition, the obtained estimate for the dimension and the size of the attractor are **independent** of the rotational inertia parameter γ, which is known to change the character of dynamics from *hyperbolic* $(\gamma > 0)$ to *parabolic like* $(\gamma = 0)$. Other properties such as additional smoothness of attractors, upper-semicontinuity with respect to parameter γ and existence of inertial manifolds are also presented.

Key Words: Von Karman evolutions, thermoelastic plates, attractors, dimension, rate of attraction.

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1. Introduction

In what follows below we shall describe model under consideration which is thermoelastic von Karman plate subjected to an external and internal forcing. Other types of nonlinearities (eg Berger's plates) can be considered as well -see [24,10,11] - however for the sake of concretness we limit ourselves to von Karman nolinearities which are representative of major mathematical difficulties encountered.

The corresponding equations (see, e.g., [43,45] and the references therein) have the following form

$$\begin{cases} u_{tt} - \gamma\Delta u_{tt} + \alpha\Delta\theta + \Delta^2 u - [u, v(u) + F_0] = p(x), \quad x \in \Omega,\, t > 0, \\ \theta_t - \eta\Delta\theta - \alpha\Delta u_t = 0, \quad x \in \Omega,\, t > 0, \end{cases} \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^2$ with the boundary $\partial\Omega = \Gamma$, Δ denotes the Laplace operator, F_0 and p are given functions with regularity specified later. Von Karman bracket $[\cdot, \cdot]$ is given by

$$[u, v] = \partial^2_{x_1} u \cdot \partial^2_{x_2} v + \partial^2_{x_2} u \cdot \partial^2_{x_1} v - 2 \cdot \partial^2_{x_1 x_2} u \cdot \partial^2_{x_1 x_2} v \ , \tag{2}$$

and Airy's stress function $v = v(u)$ is a solution to the problem

$$\Delta^2 v + [u, u] = 0, \quad v|_{\partial\Omega} = \frac{\partial v}{\partial n}\Big|_{\partial\Omega} = 0. \tag{3}$$

The temperature θ satisfies the Dirichlet boundary condition : $\theta = 0$ on Γ. The boundary conditions imposed on the displacement u are either "clamped":

$$u = \frac{\partial}{\partial\nu} u = 0 \ \text{ on } \ \Gamma, \tag{4}$$

where ν is the outer normal vector, or else "hinged (simply supported)":

$$u = \Delta u = 0 \ \text{ on } \ \Gamma. \tag{5}$$

The parameters α and η are positive and γ is non-negative. Parameter γ is proportional to the square of the thickness of the plate and in some models it is neglected (i.e. $\gamma = 0$). The case $\gamma > 0$ corresponds to taking into account rotational inertia of filaments of the plate.

The characteristics of these two models, particularly with respect to stability analysis, are very different. From physical point of view the main peculiarities of the model in (1) are (i) possibility of large deflections of the plate and (ii) small changes of the temperature near the reference temperature of the plate (which is reasonable in the absence of phase transitions). We refer to [43,45,27,38] for further discussions and references.

The main aim in this paper is to provide a survey of results pertinent to well-posedness and long time behavior of the thermal von Karman evolutions described by (1), (2). , Particular emphasis will be placed on dependence of regularity and

long time characteristics with respect to varying parameters $0 \leq \gamma \leq M_\gamma$ for some (fixed) positive constants M_γ. For simplicity we will be taking $M_\gamma = 1$. This includes questions such as:
(i) existence and uniqueness of weak solutions,
(ii) uniform stability for the unforced system,
(iii) existence of a compact global attractor and its structure,
(ii) smoothness and finite dimensionality of the attractor,
(iii) uniform decay rates to equilibria, and (iv) upper semi-continuity of family of attractors (with respect to the parameters γ and
(v) existence of inertial manifolds.

In order to point out timeliness of the topic under consideration, we wish to note that the issue of uniform decay rates for *linear*, unforced, thermoelastic plates has been settled down only recently. Indeed, results of the previous literature did require an addition of mechanical damping (boundary or interior), in order to force exponential decay rates for the energy function, see [44] and references therein. Instead, recent progress in the area of control theory and inverse problems, [1,3,8,9,36,41,43,51,54,57] has provided a stimulus to the field and produced an array of results on controllability, analyticity (when $\gamma = 0$) and uniform stability *without* any mechanical dissipation. In fact, it was shown in [2] that not only linear thermoelastic plates with either hinged or clamped boundary conditions are exponentially stable without *any mechanical dissipation*, but also that the decay rates are *independent* on the values of rotational parameter $\gamma \geq 0$.

It is a purpose of this paper to provide fairly complete description of long time behavior of thermoelastic plates driven by von Karman nonlinearity with both internal and external forcing **and** *without* **any mechanical (viscous or structural) dissipation.**

In order to gain a better understanding of the problem under consideration, one should note that topological behavior of the model is strongly dependent on the parameter $\gamma \geq 0$. It is by now well known, that the parameter γ changes drastically the linear dynamics from analytic $\gamma = 0$ to hyperbolic-like $\gamma > 0$ [53,54]. This implies that the flow has additional regularity for the limit case $\gamma = 0$, while these properties completely disappear when $\gamma > 0$. Our main challenge is to characterize long time behavior of the thermal plates, *uniformly* with respect to the values of the parameter $\gamma \geq 0$. This includes: (i) seeking an upper bound for dimensionality of attractors that are uniform in γ and κ, (ii) seeking an uniform measure of regularity enjoyed by trajectories evolving on the attractor, (iii) establishing upper semicontinuity with respect to γ.

2. Generation of a semi-flow and its properties.

2.1. Abstract form of the problem. In what follows we assume that the domain Ω is either smooth or rectangular. We denote by $H^s(\Omega)$ the L_2-based Sobolev space of the order s and by $H^s_0(\Omega)$ the closure of $C_0^\infty(\Omega)$ in $H^s(\Omega)$. We also use the following notation:

$$||u|| \equiv |u|_{L_2(\Omega)}, \quad (u,v) \equiv (u,v)_{L_2(\Omega)}, \quad \langle u,v\rangle \equiv (u,v)_{L_2(\Gamma)}, \quad \|u\|_s \equiv |u|_{H^s(\Omega)}.$$

In the space $H = L_2(\Omega)$ we define the operator A by the formula

$$Au = -\Delta u, \quad u \in \mathcal{D}(A) = H^2(\Omega) \cap H_0^1(\Omega)$$

and consider the operator $M_\gamma = I + \gamma A$. It is well-known that the both operators A and M_γ are positive self-adjoint operators in H. We also introduce the biharmonic operator

$$A_1 u = \Delta^2 u, \quad u \in \mathcal{D}(A_1) = \begin{cases} H^4(\Omega) \cap H_0^2(\Omega), & \text{(clamped b.c.);} \\ \{u \in H^4(\Omega) : u = \Delta u = 0 \text{ on } \Gamma\}, & \text{(hinged b.c.)} \end{cases}$$

In the "commutative" case of hinged boundary conditions (5) one has $A_1 = A^2$ which provides a lot of symmetry for the problem. Indeed, all the operators A, M_γ and A_1 do commute. This feature simplifies substantially the analysis with respect to *clamped* case (4), where the latter requires several additional technical estimates that account for the lack of commutativity.

We also introduce nonlinear mapping $B(\cdot)$ by the formula

$$B(u) = [u, v(u) + F_0] + p(x), \quad u \in H^2(\Omega), \tag{6}$$

where $v(u) \in H^2(\Omega)$ is determined by u via (3).

With the above notation, equations in (1) with the boundary conditions considered can be written in the form

$$\begin{cases} M_\gamma u_{tt} - \alpha A\theta + A_1 u = B(u), \\ \\ \theta_t + \eta A\theta + \alpha A u_t = 0, \end{cases} \tag{7}$$

We equip (7) with initial data $u|_{t=0} = u_0$, $u_t|_{t=0} = u_1$, $\theta|_{t=0} = \theta_0$. We note that long-time dynamics of the models in (7) with the hinged boundary conditions and $\gamma = 0$ has been studied in [19] in the context of inertial manifolds.

2.2. NONLINEAR SEMIGROUP. We begin by introducing appropriate phase (energy) spaces $\mathcal{H}_\gamma$ which capture dependence on the varying parameters γ. Our aim here is to present well-posedness of a continuous semi-flow for the models (7). By this we mean existence, uniqueness and continuous dependence of solutions with respect to initial data and $t > 0$.

For every $\gamma \geq 0$ we introduce the Hilbert space

$$\mathcal{H}_\gamma = \mathcal{D}(A_1^{1/2}) \times V_\gamma \times H \tag{8}$$

where $H = L_2(\Omega)$, $\mathcal{D}(A_1^{1/2}) = H_0^2(\Omega)$ in the clamped case and $\mathcal{D}(A_1^{1/2}) = \mathcal{D}(A) = (H^2 \cap H_0^1)(\Omega)$ in the hinged case, and $V_\gamma \equiv D(M_\gamma^{1/2})$ which is $H_0^1(\Omega)$ for $\gamma > 0$ and $L_2(\Omega)$ for $\gamma = 0$. We equip the space $\mathcal{H}_\gamma$ with the norm

$$|U|_\gamma^2 = \|A_1^{1/2} u_0\|^2 + \|M_\gamma{}^{1/2} u_1\|^2 + \|\theta\|^2, \quad U = (u_0; u_1; \theta),$$

We have that $\mathcal{D}(A_1^{1/2}) \subseteq \mathcal{D}(A)$ and $\|A_1^{1/2} u_0\| = \|Au_0\|$ for $u \in \mathcal{D}(A_1^{1/2})$.

We begin the discussion of wellposedness of weak solution by considering first *linear* problem, ie when $B(u) = 0$. In this case standard application of Lumer-Phillips Theorem yields an existence of a strongly continuous semigroup of contractions $\mathcal{S}_t^\gamma$ defined on $\mathcal{H}_\gamma$. However, the properties of this semigroup are very different for $\gamma > 0$ and $\gamma = 0$. We have

section 1.
- **$\gamma = 0$:** *In the case rotational inertia are not accounted, the semigroup $\mathcal{S}_t^0$ is analytic on $\mathcal{H}_0$. [57,54,47]*
- **$\gamma > 0$:** *In the rotational case, the semigroup $\mathcal{S}_t^\gamma$ has predominantly hyperbolic character. More specifically, it can be written as $\mathcal{S}_t^\gamma = T_t^\gamma + K_t$. where T_t^γ is a group and K_t is compact for every $t > 0$. [53,48].*

Theorem 1 remains valid also in the case of "free" boundary conditions [53,50]. Though, in this latter case the proofs are more delicate.

Wellposedness of solutions in the nonlinear case is more subtle. For the case $\gamma = 0$ the analysis of wellposedness relies on the additional regularity of the semigroup (analyticity). However, arguing this way, the estimates representing wellposedness and continuous dependence on the initial data do depend on γ. Instead, by using sharp regularity of Airy's stress function, (see [31] and also Lemma 1.2 below) this can be avoided. As the result one obtains wellposedness theory with the estimates independent on $\gamma \geq 0$.

The following well-posedness result follows from regularity of von Karman bracket (2) (see Lemma 1.2 below) along with analyticity property of the semigroup when $\gamma = 0$.

Proposition 1.1 (Well-posedness, [26]). *Assume that $F_0 \in W^2_\infty(\Omega)$ and $p \in L_2(\Omega)$. Then*

- **Existence and Uniqueness** *for all initial data $U_0 = (u_0, u_1, \theta_0) \in \mathcal{H}_\gamma$ problem (7) possesses a unique (weak) solution $U(t) \equiv (u(t), u_t(t), \theta(t)) \in C([0,\infty), \mathcal{H}_\gamma)$ which depends continuously on the initial data. This solution satisfies the energy balance equality*

$$\mathcal{E}_\gamma(u(t), u_t(t), \theta(t)) + \eta \int_s^t ||A^{1/2}\theta(\tau)||^2 d\tau = \mathcal{E}_\gamma(u(s), u_t(s), \theta(s)) \tag{9}$$

for all $t \geq s \geq 0$, where $\mathcal{E}_\gamma(u, u_t, \theta)$ is the energy functional for the model (7) given by

$$\mathcal{E}_\gamma(u, u_t, \theta) = E_{\gamma,\kappa}(u, u_t, \theta) - \frac{1}{2} \int_\Omega ([F_0, u]u + 2pu)dx \tag{10}$$

with

$$E_\gamma(u, u_t, \theta) = \frac{1}{2}\left[||Au||^2 + ||M_\gamma{}^{1/2} u_t||^2 + \frac{1}{2}||\Delta v(u)||^2 + ||\theta||^2\right]. \tag{11}$$

Moreover, when $\gamma = 0$, $U(t) \in C\left((0,T]; H^3(\Omega) \times H^1(\Omega) \times H^1(\Omega)\right)$ *for every* $T > 0$, *and*

$$\|u(t)\|_3^2 + \|u_t(t)\|_1^2 + \|\theta(t)\|_1^2 \leq \frac{C_\kappa(T,R)}{t}, \quad t \in (0,T],\ |U|_{0,\kappa} \leq R. \tag{12}$$

- **Lipschitz property** *Let* $U_1, U_2 \in \mathcal{H}_\gamma$ *and* $|U_i|_\gamma \leq R$. *Then*

$$|S_t^\gamma U_1 - S_t^\gamma U_2|_\gamma \leq e^{a_R t}|U_1 - U_2|_\gamma, \quad t > 0, \tag{13}$$

where the constant $a_R > 0$ *does not depend on* $\gamma \geq 0$

The above Proposition allows to define a strongly continuous semi-flow -semigroup S_t^γ acting on $\mathcal{H}_\gamma$. The main idea behind the proof [26] is to consider the nonlinear evolution as a locally Lipschitz perturbation of a contraction *linear* semigroup on $\mathcal{H}_\gamma$. Indeed, the nonlinear term $B(u)$ is locally Lipschitz, on the strength of regularity result given in (16). The key role in our analysis is played by the following regularity of von Karman bracket (2).

Lemma 1.2 ([25,31]). *Assume that* Ω *is either smooth, bounded domain or a rectangular domain. Let* Δ^{-2} *denotes the inverse of* Δ^2 *supplied with clamped boundary conditions. Then the bilinear map* $(u,w) \mapsto G(u,w) \equiv \Delta^{-2}[u,w]$ *is bounded from* $H^2(\Omega) \times H^2(\Omega)$ *into* $W^2_\infty(\Omega)$. *We also have the following estimates*

$$|[u,w]|_{H^{-2}(\Omega)} \leq C|u|_{H^1(\Omega)}|w|_{H^2(\Omega)}, \quad u \in H^1(\Omega),\ w \in H^2(\Omega), \tag{14}$$

$$|G(u,v)|_{W^2_\infty(\Omega)} \leq C|u|_{H^2(\Omega)}|v|_{H^2(\Omega)}, \quad u,v \in H^2(\Omega). \tag{15}$$

Consequently,

$$|[w, G(u,v)]|_{L_2(\Omega)} \leq C|u|_{H^2(\Omega)}|v|_{H^2(\Omega)}|w|_{H^2(\Omega)}. \tag{16}$$

We note that standard regularity of Airy's stress function [56]

$$|G(u,v)|_{H^{3-\epsilon}(\Omega)} \leq C|u|_{H^2(\Omega)}|v|_{H^2(\Omega)}, \quad \epsilon > 0 \tag{17}$$

will not be sufficient for most of the arguments in this paper. In fact, regularity in (17) does not imply (16), where the latter is essential for the analysis to follow.

The solutions to problem (7) generate a family of dynamical systems with the phase spaces $\mathcal{H}_\gamma$ given by (8). The evolution operator S_t^γ is given by the formula $S_t^\gamma(u_0; u_1; \theta_0) = (u(t); u_t(t); \theta(t))$, where $u(t)$ and $\theta(t)$ solve (7) with initial data in $\mathcal{H}_\gamma$. So, in all cases considered we have well defined semi-flow on the space $\mathcal{H}_\gamma$. When $\gamma > 0$, the corresponding semi-flow is predominantly hyperbolic. When $\gamma = 0$ the semi-flow is parabolic like.

In what follows below we discuss "regular solutions". The existence of such is asserted below.

Proposition 1.3 (Regular solutions, [26]). *Let* $W_\gamma = \{u \in D(A_1^{1/2}) : A_1 u \in V'_\gamma\}$, *where* V'_γ *is dual to* $V_\gamma = \mathcal{D}(M_\gamma^{1/2})$. *We equip* W_γ *with the norm* $\|A_1 \cdot\|_{V'_\gamma}$. *For the initial data such that*

$$u_0 \in W_\gamma,\ u_1 \in D(A_1^{1/2}),\ \theta_0 \in D(A), \tag{18}$$

the corresponding solutions $(u(t), \theta(t))$ *to problem* (7) *have the following regularity:*

$$(u(t), u_t(t), u_{tt}(t), \theta(t), \theta_t(t)) \in C\left(\mathbb{R}_+; W_\gamma \times D(A_1^{1/2}) \times V_\gamma \times D(A) \times L_2(\Omega)\right).$$

Moreover $w(t) = u_t(t)$ *and* $\xi(t) = \theta_t(t)$ *solves the following equations*

$$M_\gamma w_{tt} - \alpha A\xi + A_1 w = B'(u(t))w, \quad \text{and} \quad \xi_t + \eta A\xi + \alpha A w_t = 0, \tag{19}$$

with an appropriate initial data.

Since $D(A_1^{1/4}) \sim H_0^1(\Omega) \sim D(M_\gamma^{1/2})$ in the case $\gamma > 0$ [33] and thus by Closed Graph Theorem $M_\gamma^{-1/2} A_1^{1/4}$ is an isomorphism on $L_2(\Omega)$, we have that $W_\gamma = \mathcal{D}(A_1^{3/4}) \subset H^3(\Omega)$ and $V_\gamma = H_0^1(\Omega)$. In the case $\gamma = 0$ we obviously have that $W_\gamma \subset H^4(\Omega)$ and $V_\gamma = L_2(\Omega)$.

2.3. Backward uniqueness of the semi-flow. Backward uniqueness for thermoelastic nonlinear plate, beside being of interest in its own rights, arises as an issue in the context of studying properties of attractors. Indeed, it becomes a tool in proving certain characteristics of attractors. Since the thermoelastic dynamics is represented by a continuous semi-flow - and not a flow - the issue of backward uniqueness is far from obvious. When $\gamma = 0$ the analyticity of the underlying linear semigroup provides a tool (see, e.g., [37, Sect.7.3]) for the backward unique continuation. However, when $\gamma > 0$, the problem is more subtle due to parabolic-hyperbolic mixing of the dynamics. In fact, even for linear thermoelastic plates with time independent coefficients, this property has been shown only recently [55] by using complex analysis methods. Backward uniqueness, quantitatively, means that two trajectories coinciding at a given time $t > 0$ must coincide also at any earlier time. Precise formulation of the corresponding backward uniqueness result is given below.

Proposition 1.4 (Backward Uniqueness, [40,26]). *Let* $p \in L_2(\Omega)$ *and* $F_0 \in W^2_\infty(\Omega)$. *Then the following statements hold:*

- *Let* $(u^1(t), \theta^1(t))$ *and* $(u^2(t), \theta^2(t))$ *be two solutions of equations* (7) *on an interval* $[0, T]$ *such that*

$$U^i(t) \equiv (u^i(t), u^i_t(t), \theta^i(t)) \in C([0, T], \mathcal{H}_\gamma), \quad i = 1, 2.$$

  If $U^1(T) = U^2(T)$, *then* $U^1(t) = U^2(t)$ *for every* $t \in [0, T]$.

- *Let $u(t) \in C([0,T], \mathcal{D}(A_1^{1/2}))$ and $(w(t), \xi(t))$ be a solution to the linear (non-autonomous) equations* (19) *such that*

$$W(t) \equiv (w(t), w_t(t), \xi(t)) \in C([0,T], \mathcal{H}_\gamma).$$

If $W(T) = 0$, then $W(t) = 0$ for every $t \in [0,T]$.

The proof of Proposition 1.4, given in [26], is based on adaptation of technique presented in [40], where linear and unforced thermal plates with space and time dependent coefficients are considered.

Backward uniqueness is a fundamental property not only in stability theory but also in controllability theory.

2.4. STATIONARY SOLUTIONS. We introduce the set of stationary points of S_t^γ denoted by $\mathcal{N}$ (as we see below this set does not depend on γ):

$$\mathcal{N} = \{V \in \mathcal{H}_\gamma \ : \ S_t^\gamma V = V \text{ for all } t \geq 0\} \ .$$

One can see that every stationary point V has the form $V = (u, 0, 0)$ where $u = u(x) \in H^2(\Omega)$ is a weak (variational) solution to the problem

$$\Delta^2 u = [v(u) + F_0, u] + p \ \text{ in } \ \Omega,$$

with the corresponding boundary condition (either (4) or (5)), where the function $v(u)$ solves (3). In particular, stationary point do not depend on the parameters γ, α and η. One can also see that $\mathcal{N} \subset \{U \in \mathcal{H}_\gamma : |U|_\gamma \leq R_0\}$, where R_0 depends on $\|F_0\|_{W^{2,\infty}(\Omega)}$ and $\|p\|_{L_2(\Omega)}$ *only.* We use this fact in [26] to prove some uniform estimates for the attractor.

It follows from the corresponding energy relation, the full energy $\mathcal{E}_\gamma$ given by (10)) is non-increasing. Therefore the set

$$\mathcal{E}_R^\gamma = \left\{U = (u_0; u_1; \theta_0) \in \mathcal{H}_\gamma \ : \ \mathcal{E}_\gamma(u_0, u_1, \theta_0) \leq R^2\right\} \tag{20}$$

is forward invariant for every $R > 0$, i.e., $S_t^\gamma \mathcal{E}_R^\gamma \subset \mathcal{E}_R^\gamma$ for $t \geq 0$. One can also see, because of the topological equivalence between the norm induced by the energy and the topology of $\mathcal{H}_\gamma$ that there exists $R_0^* \geq R_0$ which depends on $\|F_0\|_{W^{2,\infty}(\Omega)}$ and $\|p\|_{L_2(\Omega)}$ *only* such that $\mathcal{N} \subset \mathcal{E}_{R_0^*}^\gamma$. As we see below this property makes it possible to prove that the global attractor belongs to the set $\{U \in \mathcal{H}_\gamma : |U|_\gamma \leq R_*\}$, where R_* depends on $\|F_0\|_{W^{2,\infty}(\Omega)}$ and $\|p\|_{L_2(\Omega)}$ only.

3. Attractors for abstract dynamical systems

We recall (see, e.g., [6,17,65]) that by definition a global attractor for a dynamical system (X, S_t) on a complete metric space X is a closed bounded set $\mathcal{A}$ in X which is invariant (i.e. $S_t\mathcal{A} = \mathcal{A}$ for any $t > 0$) and uniformly attracting, i.e.

$$\lim_{t\to+\infty} \sup_{y\in B} \operatorname{dist}_X\{S_t y, \mathcal{A}\} = 0 \quad \text{ for any bounded set } \quad B \subset X.$$

Remark 2. It follows directly from the definition of that a global attractor for (X, S_t) is a collection of all bounded full trajectories of the semi-flow S_t. We recall the a continuous curve $\gamma = \{u(t) : t \in \mathbb{R}\}$ in X is said to be a *full trajectory*, if $S_t u(\tau) = u(t+\tau)$ for all $t \geq 0$ and $\tau \in \mathbb{R}$. We will use this simple observation in the study of continuity properties of attractors with respect to parameters.

Let $\mathcal{N}$ be the set of stationary points of the dynamical system (X, S_t), i.e.

$$\mathcal{N} = \{v \in X \ : \ S_t v = v \text{ for all } t \geq 0\}.$$

We define the *unstable manifold* $\mathcal{M}^u(\mathcal{N})$ emanating from the set $\mathcal{N}$ as a set of all $y \in X$ such that there exists a full trajectory $\gamma = \{u(t) : t \in \mathbb{R}\}$ with the properties $u(0) = y$ and $\operatorname{dist}_X(u(t), \mathcal{N}) \to 0$ as $t \to -\infty$. It is clear that $\mathcal{M}^u(\mathcal{N})$ is an invariant set. It is also easy to prove (see, e.g., [6], [17] or [65]) that if the dynamical system (X, S_t) possesses a global attractor $\mathcal{A}$, then $\mathcal{M}^u(\mathcal{N}) \subset \mathcal{A}$. For gradient systems it is possible to prove that $\mathcal{M}^u(\mathcal{N}) = \mathcal{A}$. We give the following definition (see [6,17,34,42,65]).

Definition 2.1. *A dynamical system (X, S_t) is said to be* gradient *if it possesses a strict Lyapunov function, i.e. there exists a continuous functional $\Phi(y)$ defined on X such that (i) the function $t \mapsto \Phi(S_t y)$ is nonincreasing for any $y \in X$, and (ii) the equation $\Phi(S_t y) = \Phi(y)$ for all $t > 0$ and for some $y \in X$ implies that $S_t y = y$ for all $t > 0$, i.e. y is a stationary point of (X, S_t).*

It follows from energy relation (9) that the the energy $\mathcal{E}_\gamma(u, u_t, \theta)$ is a strict Lyapunov function for the dynamical system $(\mathcal{H}_\gamma, S_t^\gamma)$. Thus this system is gradient.

We have the following result on the structure of a global attractor (for the proof we refer to any book from the list [6,17,34,42,65]).

section 3. *Let a gradient dynamical system (X, S_t) possess a compact global attractor $\mathcal{A}$. Then $\mathcal{A} = \mathcal{M}^u(\mathcal{N})$. Moreover the global attractor $\mathcal{A}$ consists of full trajectories $\gamma = \{u(t) : t \in \mathbb{R}\}$ such that*

$$\lim_{t \to -\infty} \operatorname{dist}_X(u(t), \mathcal{N}) = 0 \ \textit{and} \ \lim_{t \to +\infty} \operatorname{dist}_X(u(t), \mathcal{N}) = 0. \tag{21}$$

The following assertion describes long-time behavior of individual trajectories (for the proof we refer to [17] or [65], for instance).

section 4. *Assume that a gradient dynamical system (X, S_t) possesses a compact global attractor $\mathcal{A}$. Then for any $x \in X$ we have $\lim_{t\to+\infty} \operatorname{dist}_X(S_t x, \mathcal{N}) = 0$, i.e. any trajectory stabilizes to the set $\mathcal{N}$ of stationary points.*

Theorems 3 and 4 imply the following assertion.

Corollary 4.1. *Assume that a gradient dynamical system (X, S_t) possesses a compact global attractor $\mathcal{A}$ and $\mathcal{N} = \{e_1, \ldots, e_n\}$ is a finite set. Then $\mathcal{A} = \cup_{i=1}^n \mathcal{M}^u(e_i)$, where $\mathcal{M}^u(e_i)$ is the unstable manifold of the stationary point e_i, and*

(i) *the global attractor $\mathcal{A}$ consists of full trajectories $\gamma = \{u(t) : t \in \mathbb{R}\}$ connecting pairs of stationary points, i.e. any $u \in \mathcal{A}$ belongs some full trajectory γ and for any $\gamma \subset \mathcal{A}$ there exists a pair $\{e, e^*\} \subset \mathcal{N}$ such that $u(t) \to e$ as $t \to -\infty$ and $u(t) \to e^*$ as $t \to +\infty$;*

(ii) *for any $v \in X$ there exists a stationary point e such that $S_t v \to e$ as $t \to +\infty$.*

The following assertion provides exponential rate of stabilization to the attractor along with some additional properties of the attractor (see, e.g., [6], [34] and also Theorems 4.7 and 4.8 in the survey [62]).

section 5. *In addition to previous hypotheses, asssume that (i) an evolution operator S_t is C^1, (ii) the set $\mathcal{N}$ of equilibrium points is finite and all equilibria are hyperbolic, and (iii) there exists a Lyapunov $\Phi(x)$ function such that $\Phi(S_t x) < \Phi(x)$ for all $x \in X$, $x \notin \mathcal{N}$ and for all $t > 0$. Then*

- *For any $y \in X$ there exists $e \in \mathcal{N}$ such that*
$$\|S_t y - e\|_X \le C_y e^{-\omega t}, \quad t > 0.$$
Moreover, for any bounded set B in X we have that
$$\sup\{\mathrm{dist}\,(S_t y, \mathcal{A}) \,:\, y \in B\} \le C_B e^{-\omega t}, \quad t > 0. \tag{22}$$
Here above $\mathcal{A}$ is a global attractor, C_y, C_B and ω are positive constants, ω in (22) depends on the minimum, over $e \in \mathcal{N}$, of the distance of the spectrum of $D[S_1 e]$ to the unit circle in $\mathbb{C}$.

Asymptotic smoothness is the most critical property which is necessary for the existence of a compact global attractor. There are several approaches to the proof of this property. For instance, we can use either a splitting method (see [6,17,34,65] and the references therein) or the method of energy type identities (see [7] and also the survey [62]). However the stabilizability estimate which we prove in [26] makes it possible to apply the following criterium (see [12,34] and also [24] for some generalizations) for the proof of asymptotic smoothness of the dynamical system $(\mathcal{H}_\gamma, S_t^\gamma)$ generated by (1).

section 6. *Let (X, S_t) be a dynamical system on a complete metric space X endowed with a metric d. Assume that for any bounded positively invariant set B in X there exist numbers $T > 0$ and $0 < q < 1$, and a pseudometric ϱ_B^T on $C(0, T; X)$ such that*

(i) *the pseudometric ϱ_B^T is precompact (with respect to X) in the following sense: any sequence $\{x_n\} \subset B$ has a subsequence $\{x_{n_k}\}$ such that the sequence $\{y_k\} \subset C(0, T; X)$ of elements $y_k(\tau) = S_\tau x_{n_k}$ is Cauchy with respect to ϱ_B^T;*

(ii) *the following inequality holds*
$$d(S_T y_1, S_T y_2) \le q \cdot d(y_1, y_2) + \varrho_B^T(\{S_\tau y_1\}, \{S_\tau y_2\}), \tag{23}$$
for every $y_1, y_2 \in B$, where we denote by $\{S_\tau y_i\}$ the element in the space $C(0, T; X)$ given by function $y_i(\tau) = S_\tau y_i$.

Then (X, S_t) is an asymptotically smooth dynamical system.

An important characteristic of a global attractor is its (fractal) dimension. We recall that the *fractal dimension* $\dim_f^X M$ of a compact set M in a complete metric space X is defined by

$$\dim_f^X M = \limsup_{\varepsilon \to 0} \frac{\ln N(M, \varepsilon)}{\ln(1/\varepsilon)} ,$$

where $N(M, \varepsilon)$ is the minimal number of closed sets in X of the diameter 2ε which cover the set M. We note that fractal ($\dim_f^X M$) and Hausdorff ($\dim_H^X M$) dimensions satisfies the inequality $\dim_f^X M \geq \dim_H^X M$. Thus the finiteness of $\dim_H^X M$ implies the finiteness of the Hausdorff dimension and lower bounds for $\dim_H^X M$ provide us with lower bounds for the fractal dimension.

Our proof of finite dimensionality of the attractors for $(\mathcal{H}_\gamma S_t^\gamma)$ is based on the following assertion (see [24] and also [21,22] which contain other versions of the theorem stated below).

section 7. *Let X be a Banach space and M be a bounded closed set in X. Assume that there exists a mapping $V : M \mapsto X$ such that $M \subseteq VM$ and also*

(i) V is Lipschitz on M, i.e, there exists $L > 0$ such that

$$\|Vv_1 - Vv_2\| \leq L\|v_1 - v_2\|, \quad v_1, v_2 \in M;$$

(ii) there exist compact seminorms $n_1(x)$ and $n_2(x)$ on X such that

$$\|Vv_1 - Vv_2\| \leq \eta\|v_1 - v_2\| + K \cdot [n_1(v_1 - v_2) + n_2(Vv_1 - Vv_2)]$$

for any $v_1, v_2 \in M$, where $0 < \eta < 1$ and $K > 0$ are constants (a seminorm $n(x)$ on X is said to be compact iff for any bounded set $B \subset X$ there exists a sequence $\{x_n\} \subset B$ such that $n(x_m - x_n) \to 0$ as $m, n \to \infty$).

Then M is a compact set in X of a finite fractal dimension. Moreover, we have the estimate

$$\dim_f^X M \leq \left[\ln \frac{2}{1+\eta}\right]^{-1} \cdot \ln m_0 \left(\frac{4K(1+L^2)^{1/2}}{1-\eta}\right),$$

where $m_0(R)$ is the maximal number of pairs (x_i, y_i) in $X \times X$ possessing the properties

$$\|x_i\|^2 + \|y_i\|^2 \leq R^2, \quad n_1(x_i - x_j) + n_2(y_i - y_j) > 1, \quad i \neq j.$$

4. Asymptotic behavior of von Karman thermal plates

4.1. Exponential decays to a single equilibrium. We begin by recalling uniform stability results in the case when the attractor is trivial and consists just of one point. Wlog we assume that the only equilibrium is zero, so we take $F_0 = 0, p = 0$. In that case we have

section 8. *Let $F_0 = 0, p = 0$. Then the energy of the nonlinear plate decays to zero exponenntially, with the rates independent on $0 \le \gamma \le 1$. This is to say there exists constant $\omega > 0$ such that*

$$E_\gamma(u(t), u_t(t), \theta(t)) \le E_\gamma(u(0), u_t(0), \theta(0))e^{-\omega t},$$

where the energy functional $E_\gamma(u, u_t, \theta)$ is given by (11).

Exponential decay rates presented in theorem 8 were established in [2] -for the linear case and in [4,5] -for the nonlinear case. We also note that the same result holds for "free" boundary conditions-though the proof is much more technical [3]. In the case $\gamma = 0$ thermal plates with hinged boundary conditins have been known for some time [41,38] to be exponentially decaying.

Other related results on exponential stability of nonlinear thermal plates can be found in [51,8,4,5]

4.2. GLOBAL ATTRACTORS. Our main results on global attractors for dynamical systems $(\mathcal{H}_\gamma, S_t^\gamma)$ with $0 \le \gamma \le 1$ are formulated below.

section 9 (Compact Attractors). *For every $0 \le \gamma \le 1$ the dynamical system $(\mathcal{H}_\gamma, S_t^\gamma)$ is gradient and possesses a compact global attractor $\mathcal{A}^\gamma = M_\gamma^u(\mathcal{N})$, where $M_\gamma^u(\mathcal{N})$ is unstable manifold emanating from the set $\mathcal{N}$ of stationary points. Thus the conclusions of Theorem 3 and Theorem 4 hold true for $(\mathcal{H}_\gamma, S_t^\gamma)$. Moreover,*

- **Finite-dimensionality:** *there exists $d_0 > 0$ independent of γ such that fractal dimension of $\mathcal{A}^\gamma$ in $\mathcal{H}_\gamma$ admits the estimate $\dim_f^{\mathcal{H}_\gamma} \mathcal{A}^\gamma \le d_0$ for $0 \le \gamma \le 1$.*
- **Regularity:** *any full trajectory $\{U(t) : t \in \mathbb{R}\}$ from the attractor possesses the properties*

$$||Au(t)||^2 + ||M_\gamma^{1/2} u_t(t)||^2 + ||\Delta v(u(t))||^2 + ||\theta(t)||^2 \le R_1^2 \tag{24}$$

and

$$\|u(t)\|_3^2 + ||Au_t(t)||^2 + ||M_\gamma^{1/2} u_{tt}(t)||^2 + ||\theta_t(t)||^2 + ||A\theta(t)||^2 \le R_2^2 \tag{25}$$

for all $t \in \mathbb{R}$, where the both constants R_1 and R_2 do not depend on $0 \le \gamma \le 1$ and R_1 is also independent of η and α and in the case $\gamma = 0$ we additionally have that $\|u(t)\|_4 \le R_2$ for $t \in \mathbb{R}$);

- **Upper semi-continuity:** *the family of the attractors $\mathcal{A}^\gamma$ is upper semi-continuous with respect to γ in the sense that for any $\gamma_0 \ge 0$ we have that*

$$\lim_{\gamma \to 0} \sup \left\{ \mathrm{dist}_{\mathcal{H}_{\gamma_0}} (U, \mathcal{A}^{\gamma_0}) : U \in \mathcal{A}^\gamma \right\} = 0. \tag{26}$$

We note that in the case of *isothermal* von Karman plate upper semi-continuity of the attractor when $\gamma \to 0$ was proved in [16].

Our next result relies on Theorem 5 and deals with the case when the set $\mathcal{N}$ is finite and every stationary point is hyperbolic.

section 10 (Exponential Attractor). *Assume that $\mathcal{N} = \{E_i : i = 1, \ldots, n\}$ is a finite set. Then the conclusions of Corollary 4.1 holds true for the system $(\mathcal{H}_\gamma, S_t^\gamma)$ for every $\gamma \geq 0$ In particular, $\mathcal{A}^\gamma = \cup_{i=1}^n \mathcal{M}^u(E_i)$. Moreover, if every stationary point $E_i = (e_i; 0; 0)$ is hyperbolic in the sense that the equation $A_1 w = B'(e_i)w$, where $B'(u)$ is Frechet derivative of the mapping B given by (6), has only trivial solutions. Then:*

- *For any $U_0 \in \mathcal{H}_\gamma$, there exists an equilibrium point $E = (e, 0, 0) \in \mathcal{H}_\gamma$ and constants $\omega > 0$ and $C_{U_0} > 0$ (possibly depending on γ) such that*
$$|S_t^\gamma U_0 - E|_\gamma \leq C_{U_0} e^{-\omega t}, \quad t > 0.$$
Moreover, for any bounded set B in $\mathcal{H}_\gamma$ we have that
$$\sup \{\mathrm{dist}\,(S_t^\gamma, \mathcal{A}^\gamma) \,:\, U \in B\} \leq C_B e^{-\omega t}, \quad t > 0. \tag{27}$$
Here $\mathcal{A}^\gamma$ is the global attractor, C_B and ω are positive constants which may depend on γ.

- *For each $E \in \mathcal{N}$ the unstable manifold $\mathcal{M}^u(E)$ is an embedded C^1-submanifold of $\mathcal{H}_\gamma$ of finite dimension* $\mathrm{ind}\,(E)$*, which implies that*
$$\dim_f \mathcal{A}^\gamma \geq \dim_H \mathcal{A}^\gamma = \max_{E \in \mathcal{N}} \mathrm{ind}\,(E). \tag{28}$$

Remark 11. The first statement of Theorem 10 implies that the global attractor is exponential. However this property requires finiteness and hyperbolicity of the set $\mathcal{N}$ of equilibria. Whether the dependence of exponential rate of attraction in (27) on $\gamma \geq 0$ could be surpressed, is not known at the present time. We also note that in the general (non-hyperbolic) case one can apply Corollary 2.23 [24] and argument similar given in the proof of Theorem 4.43 [24] to obtain the existence of *exponential fractal attractor* (inertial set) with an uniform (with respect to γ) estimate for the dimension. For details concerning a general notion of an exponential fractal attractor we refer to the monograph [29].

Remark 12. If we compare (28) with the result on the dimension from Theorem 9, then we obtain that $\max_{E \in \mathcal{N}} \mathrm{ind}\,(E)$ can be estimated from above by a constant independent of γ.

Remark 13. We note that the present treatment does not rely on analyticity of the semigroup associated with the model when $\gamma = 0$. All the estimates obtained for the size and the dimension of the attractor are independent on $\gamma \geq 0$. This was possible to achieve for both simply supported and clamped boundary conditions. However, in the case of free boundary conditions, the situation is more complicated. To our best knowledge, there are no appropriate estimates -independent on γ even in the linear case. Nevertheless, the methods of the paper [26] provide all the results on attractors for each value of the parameter ($\gamma > 0$ and $\gamma = 0$). In the case $\gamma = 0$, critical use of the analyticity (see, e.g., [54]) of the semigroup will have to play the role. How to make these estimates (in the case of free boundary conditions) uniform with respect to γ is an open problem.

4.3. INERTIAL MANIFOLDS. For plates with hinged boundary conditions and special geometry of the domain Ω one can prove [19] existence of inertial manifolds. We begin by recalling definition of inertial manifold.

Definition 13.1. *Let $\mathcal{M}$ be a finite-dimensional surface in $\mathcal{H}$ of the following structure:*

$$\mathcal{M} \equiv \{p + \Phi)p), p \in \mathcal{P}\mathcal{H}, \Phi : \mathcal{P}\mathcal{H} \to (I - \mathcal{P})\mathcal{H}\} \tag{29}$$

where $\mathcal{P}$ is a finite dimensional projector and Φ is a Lipschitz continuous mapping. Then, $\mathcal{M}$ is said to be an inertial manifold for the dynamical system $(S_t, \mathcal{H})$, iff (i) the surface $\mathcal{M}$ is invariant under the flow, (ii) $\mathcal{M}$ is exponentially attracting.

In the case of locally Lipschitz nonlinearities, a locally invariant manifold is relevant. This means that the invariance property is restricted to some ball in $\mathcal{H}$.

Definition 13.2. *The Lipschitz surface $\mathcal{M}$ is said to be locally invariant inertial manifold, if it is exponentially attracting and, moreover, there exists $R > 0$ such that the ball B_R in $\mathcal{H}$ is absorbing, and $\mathcal{M}$ is locally invariant in B_R. This is to say, for all $u \in B_R \cap \mathcal{M}, S_t u \in B_R$ for $t \in [0, T]$, we have that $S_t u \in \mathcal{M}, t \in [0, T]$.*

The general theory of inertial manifolds was started with the paper [32] and has been developed and widely studied for deterministic systems by many authors (see, e.g., the monographs [17,28,65] and the references therein). All known results concerning existence of inertial manifolds require some gap condition on the spectrum of the linearized problem.

In the case when Ω is a rectangle, $\gamma = 0$, and the boundary conditions associated are hinged, an existence of inertial manifold has been established in [19]. This result is reported below.

We recall that the abstract form of the thermoelastic system with hinged boundary conditions is written as

$$u_{tt} - \alpha A\theta + A^2 u = B(u), \quad \theta_t + \eta A\theta + \alpha A u_t = 0 \tag{30}$$

An important role in this result is played by the properties of the roots of characteristic equation

$$z^3 - \eta z^2 + (1 + \alpha)z - \eta = 0$$

This equation has one positive root z_1 and the two remaining, z_2 and z_3, are complex conjugates.

section 14. *Consider (30) where $\Omega = (0, l_1) \times (0, l_2)$ with $p \in L_2(\Omega)$ and $F_0 \in W^{2,\infty}(\Omega)$. We assume*

1. *$\frac{l_1}{l_2}$ is rational*
2. *$\frac{1}{3} < \frac{1+\alpha^2}{\eta^2} < \infty$*
3. *either $\frac{Re\ z_2}{z_1}$ is rational , or else α is sufficiently large.*

Then the flow S_t corresponding to (30) and defined on $\mathcal{H} = D(A) \times L_2(\Omega) \times L_2(\Omega)$ possesses a locally invariant inertial manifold.

The proof of Theorem 14, given in [19], is based on spectral analysis of the linear problem. The key element is to show that certain gap condition between eigenvalues separating stable and unstable manifolds is satisfied. To accomplish this, number theoretic properties are exploited. Application of these necessitates imposition of geometric conditions listed in the theorem. Whether the same result holds in a broader context (e.g., for the case $\gamma > 0$, and/or for non-rectangle domains, or else with other boundary conditions, etc.) remains an open problem.

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